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FINALTERM EXAMINATION 2009
(Session - 1)

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ……. At x_0

► Minima
► Maxima
► None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ……. At x_0

► Minima
► Maxima
► None of these

Question No: 3 (Marks: 1) - Please choose one

$\lim_{x \to 0} \frac{\sin 2x}{x} = -----$

► 2
► 4
► 1
► ∞

Question No: 4 (Marks: 1) - Please choose one

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$$\lim_{x\to 0^+} \frac{\ln x}{1/x} = -----$$

- 1
- 0
- e
- None of these

Question No: 5 (Marks: 1) - Please choose one

$$\frac{d(\tan x)}{dx} =$$

- $\sec x$
- $\sec^2 x$
- $co\sec x$
- $co\sec^2 x$

Question No: 6 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- 0
- $\frac{-1}{x^2}$
- $\frac{4}{x^2}$
- $\frac{-4}{x^2}$

Question No: 7 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$$\frac{d}{dx}\left((c)\{h(x)\}\right) = _____$$

- 0
- $\frac{d}{dx}\left(h(x)\right)$

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- ▶ $\frac{d}{dx}(h(cx))$
- ▶ $c\frac{d}{dx}(h(x))$

Question No: 8 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$\frac{d}{dx}[f][g] =$

- ▶ $\frac{[f'][g]-[f][g']}{g^2}$
- ▶ $[f'][g']$
- ▶ $[f'][g]+[f][g']$
- ▶ $[f'][g]-[f][g']$

Question No: 9 (Marks: 1) - Please choose one

$\frac{d}{dx}[x^n] = nx^{n-1}$

The power rule, holds if n is _____

- ▶ An integer
- ▶ A rational number
- ▶ An irrational number
- ▶ All of the above

Question No: 10 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then which of the following statement is correct?

- ▶ f is a decreasing function
- ▶ f is an increasing function
- ▶ f is a constant function

Question No: 11 (Marks: 1) - Please choose one

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If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- f is concave up on (a, b).
- f is concave down on (a, b)
- f is linear on (a, b).

Question No: 12 (Marks: 1) - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

What does 'n' represent in Riemann Sum ?

- No. of Circles
- No. of Rectangles
- No. of Loops
- No. of Squares

Question No: 13 (Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x\to-\infty} f(x) = +\infty$ *and* $\lim_{x\to+\infty} f(x) = +\infty$ then f has _____ on $(-\infty, +\infty)$

- maximum value but no minimum
- minimum value but no maximum
- both maximum and minimum value

Question No: 14 (Marks: 1) - Please choose one

$$\int_2^t \frac{x^2}{2} dx$$

The expression , represents a function of :

- t
- x
- 2
- Both t and x

Question No: 15 (Marks: 1) - Please choose one

$\int cf(x)dx =$ _____

if c is a constant

- 0

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- c
- $\int f(cx)dx$
- $c\int f(x)dx$

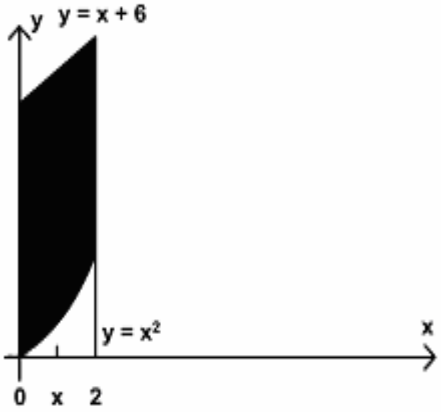
Question No: 16 (Marks: 1) - Please choose one

Sigma notation is represented by which of the following Greek letter?

- χ
- η
- Σ
- ψ

Question No: 17 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**



- $y = x + 6$
- $y = x^2$
- $x = 2$
- $x = 0$

Question No: 18 (Marks: 1) - Please choose one

In the following figure, the area bounded on the sides by the lines are **:**

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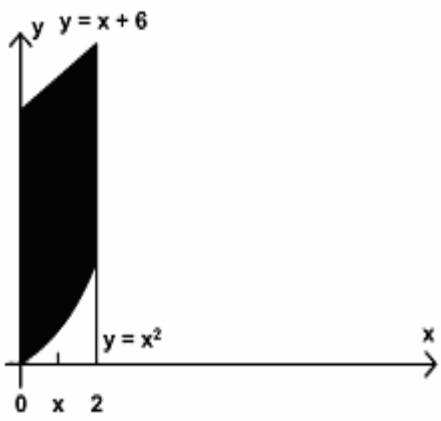
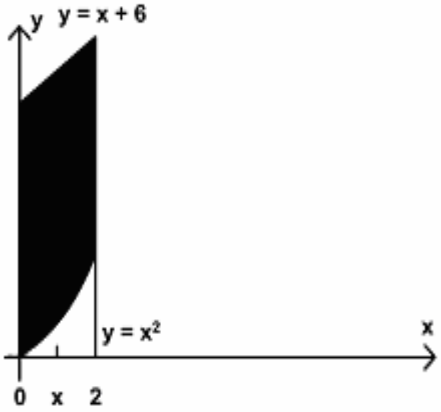


- $x = 0$
- $x = 2$
- $x = 0 \; and \; x = 2$
- $x = 6$

Question No: 19 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



- $A = \int_{0}^{2} \left[(x+6) - (x^2) \right] dx$
- $A = \int_{x}^{2} \left[(x+6) - (x^2) \right] dx$
- $A = \int_{0}^{2} \left[(x+6) + (x^2) \right] dx$
-

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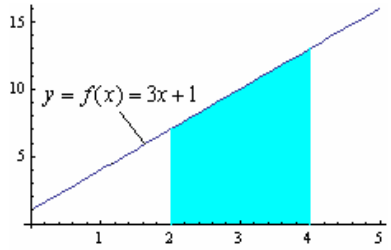
$$A = \int_0^x \left[(x+6) - (x^2) \right] dx$$

▶

Question No: 20 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y = f(x) = 3x + 1$ over the interval $[2, 4]$, evaluated by using the formula

$$Area = f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta x$$



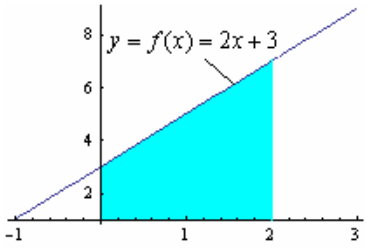
If the interval $[2, 4]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are left endpoint of each sub-interval.

▶ 17
▶ 20
▶ 23

Question No: 21 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y = f(x) = 2x + 3$ over the interval $[0, 2]$, evaluated by using the formula

$$Area = f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta x$$



If the interval $[0, 2]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are right endpoint of each sub-interval.

▶ 8
▶ 10
▶ 12

Question No: 22 (Marks: 1) - Please choose one

$If\ x > 0\ then\ \frac{d}{dx}[\ln x] =$ _____

▶ 1
▶ x
▶ $\frac{1}{x}$

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▶ $\ln \frac{1}{x}$

Question No: 23 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on [a,b] and c is a constant, then $\int_a^b c\,[f(x)+g(x)]\,dx =$ _____

▶ $\int_a^b f(cx)dx+\int_a^b g(cx)dx$

▶ $\int_a^b f(x)\,dx+\int_a^b g(x)]\,dx$

▶ $c\int_a^b f(x)dx+c\int_a^b g(x)dx$

▶ 0

Question No: 24 (Marks: 1) - Please choose one

If the function f is continuous on [a,b] and if $f(x)\geq 0$ for all x in [a,b], then which of the following gives area under the curve $y=f(x)$ over the interval [a,b]?

$\lim_{x\to\infty}\sum_{k=1}^{n}[x_k][f(x_k)]$ *where n is number of subdivisions of* $[a,b]$

▶ $\int_a^b f(x)\,dx$

▶ $\pi[radius]^2$

▶ (Width) (Height)

Question No: 25 (Marks: 1) - Please choose one

Let region R in the first quadrant enclosed between $y=3x$ *and* $y=2x^2$ is revolved about the
x-axis .Which of the following equation gives the volume of a solid by cylindrical shells?

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$$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$$

▶

$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$$

▶

$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

▶

$$V = \int_{-1}^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

▶

Question No: 26 (Marks: 1) - Please choose one

Let f is a smooth function on [a, b]. What will be the arc length L of the curve y = f(x) from x = a to x = b?

$$L = \int_b^a \sqrt{1 + [f'(x)]}dy$$

▶

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}dx$$

▶

$$L = \int_0^a \sqrt{1 + [f'(x)]}dy$$

▶

$$L = \int_a^b \sqrt{1 + [f'(x)]}dx$$

▶

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Question No: 27 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the integral

$$\int_a^b f(x)dx = \lim_{l \to a} \int_l^b f(x)dx$$

defined by is called **:**

- ► Improper
- ► Proper
- ► Line

Question No: 28 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as**:**

- ► Increasing
- ► Decreasing
- ► Nondecreasing
- ► Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} < 1$ then the sequence is known as**:**

- ► Increasing
- ► Decreasing
- ► Nondecreasing
- ► Nonincreasing

Question No: 30 (Marks: 1) - Please choose one

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx$$

Consider the indefinite integral

Let $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

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$$\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx = \int \frac{1}{t}\,dt$$

► Yes
► No

Question No: 31 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$ if $\rho = 1$, then which of the following is true?

► Converges
► Diverges
► May converges or diverges
► Gives no information

Question No: 32 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

► Converges
► Diverges
► May converges or diverges
► Gives no information

Question No: 33 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} |u_k| = |u_1| + |u_2| + |u_3| + \ldots |u_k| + \ldots$$

If the series converges , then which of the

$$\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \ldots u_k + \ldots$$

following is true for ?

► Converges
► Diverges

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► Gives no information

Question No: 34 (Marks: 1) - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that $\rho = \lim_{k\to\infty} \frac{|u_{k+1}|}{|u_k|}$ if $\rho = +\infty$, then which of the following is true?

► Then the series $\sum u_k$ diverges

► The series $\sum u_k$ converges absolutely and therefore converges

► May converges or diverges

► Gives no information

Question No: 35 (Marks: 1) - Please choose one

$\int_{-1}^{1} (x-1)\,dx = $ _____

► -2

► 0

► 2

► 4

Question No: 36 (Marks: 1) - Please choose one

How many critical points exist for a function f if $f'(x) = (x-3)(x-2)$

► Zero

► One

► Two

► Four

Question No: 37 (Marks: 1) - Please choose one

$\log_b ac =$ _____

► $\log_b a + \log_b c$

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- $\log_b a - \log_b c$
- $\dfrac{\log_b a}{\log_b c}$
- $(\log_b a)(\log_b c)$

Question No: 38 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

- $a \log_b r$
- $r \log_b a$
- $\dfrac{\log_b a}{\log_b r}$
- $\log_b a + \log_b r$

Question No: 39 (Marks: 1) - Please choose one

Let $y = \dfrac{2\sqrt{2}}{3} x^{\frac{3}{2}}$; $0 \le x \le 2$ then which of the following is the length of the curve?

$$L = \int_0^2 \sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}}\right)\right]^2} \, dx$$

- $L = \int \sqrt{1 + \left[\dfrac{d}{dx}\left(\dfrac{2\sqrt{2}}{3} x^{\frac{3}{2}}\right)\right]^2} \, dx$
- $L = \int_0^2 \sqrt{1 + \left[\dfrac{d}{dx}\left(\dfrac{2\sqrt{2}}{3} x^{\frac{3}{2}}\right)\right]^2} \, dx$

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$$L=\int_0^2\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]}dx$$

▶

Question No: 40 (Marks: 1) - Please choose one

Which of the following are *first two* terms for the Taylor series of $f(x)=e^{-x}$ at $x = 0$?

$1+(1)(x-0)$

▶

$1+(-1)(x+0)$

▶

$1+(-1)(x-0)$

▶

$(-1)(x-0)$

▶

Question No: 41 (Marks: 2)

$$\int_2^3(1-x)dx$$

Evaluate the integral

$$\int_2^3(1-x)dx$$

$$=|\,x-x^2\frac{1}{2}\,|_2^3$$

$$=\frac{1}{2}|\,2x-x^2\,|_2^3$$

$$=\frac{1}{2}(2(3-2)-(3-2)^2)$$

$$=\frac{1}{2}(2-1)$$

$$=\frac{1}{2}$$

Question No: 42 (Marks: 2)

$$\int_2^{+\infty}\frac{dx}{x^2}$$

Evaluate the improper integral

Question No: 43 (Marks: 2)

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A function $f(x) = x^2 - 4x - 9$ has critical point 2 in an interval [0, 5]. Find the maximum value of the function and point having this value.

Question No: 44 (Marks: 3)

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Evaluate:

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2 \quad , \quad x > 0$, and bounded on the sides by the lines $y = 1$ and $y = 4$

$y = x^2 \quad , \quad x > 0$

So we have

$$A = \int_1^4 x^2 dx$$

$$= | \frac{x^3}{3} |_1^4$$

$$= \frac{1}{3}(4-1)^3$$

$$= \frac{1}{3}(3)^3$$

$$= 9$$

Question No: 46 (Marks: 3)

Determine whether the following sequence converges or diverges. If it converges, find the limit.

$$\lim_{n \to \infty} \frac{5n^2 - 1}{20n + 7n^2}$$

Question No: 47 (Marks: 5)

Use the Alternating series Test to determine whether the given series converges

$$\sum_1^\infty \frac{(-1)^{n-1} \cdot n!}{2^n}$$

Question No: 48 (Marks: 5)

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Evaluate the integral

$$\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2} dt$$

Solution

$$\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2} dt$$

$$u = 2t$$

$$\frac{du}{dt} = 2dt$$

$$du = 2dt$$

so

$$= \frac{1}{4}\int_{\frac{\pi}{2}}^{0} 1+\cos u du$$

$$= \frac{1}{4} | u + \sin u |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4} | 2t + \sin 2t |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4}(2\frac{\pi}{2} + \sin 2\frac{\pi}{2})$$

$$= \frac{1}{4}(\pi + \sin \pi)$$

$$= \frac{1}{4}(\pi + 0)$$

$$= \frac{\pi}{4}$$

Question No: 49 (Marks: 5)

Evaluate the sums

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$$\sum_{k=1}^{5} k(3k+5)$$
$$= 1(3+5)+2(6+5)+3(9+5)+4(12+5)+5(15+5)$$
$$= 8+22+3(45)+4(60)+5(75)$$
$$= 8+22+135+240+375$$
$$= 780$$

Question No: 50 (Marks: 10)

Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x – axis.

$y = 1+x^3$, $x = 1, x = 2, y = 0$

from $V=\int_a^b \pi[f(x)]^2 dx$

$$V=\int_1^2 \pi[1+x^3]^2 dx$$
$$V=\int_1^2 \pi[1+x^5+2x^3]dx$$
$$V=\pi\int_1^2 (1+x^5+2x^3)dx$$
$$V=\pi\,|\,(x+\frac{1}{6}x^6+\frac{1}{2}x^4)\,|_1^2$$
$$V=\pi((2-1)+\frac{1}{6}(2-1)^6+\frac{1}{2}(2-1)^4)$$
$$V=\pi\{((2-1)+\frac{1}{6}(2-1)^6+\frac{1}{2}(2-1)^4)\}$$
$$V=\pi((1)+\frac{1}{6}+\frac{1}{2}))$$
$$V=\frac{\pi(6+1+3)}{6}$$
$$V=\frac{\pi(10)}{6}=\pi\frac{5}{3}$$

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